LETTERS TO THE EDITOR

To the editor:

The interesting article on bubble formation at vibrated orifices by Barker and de Nevers [*AIChE J.*, **30**, 37 (1984)] prompts some comments, as follows:

Description of bubble formation at upward-facing circular orifices usually distinguishes between three regimes, *vide* Cliff et al. (1978):

(a) low gas flowrates, with surface tension and gravity effects dominant;
(b) liquid viscosity and gravity effects dominant, intermediate gas flowrates;
(c) high gas flowrates, with inertia and gravity effects dominant.

In general terms, the relations between bubble volume V_B, gas flow rate $\dot{V}$ and gravity acceleration g are:

$$\text{Regime } (a)\ V_B = K_a g^{-1} \qquad (1.a)$$

$$\text{Regime } (b)\ V_B = K_b \dot{V}^{3/4} g^{-3/4} \qquad (1.b)$$

$$\text{Regime } (c)\ V_B = K_c \dot{V}^{6/5} g^{-3/5} \qquad (1.c)$$

Vibration introduces additional forces, with amplitude a, angular frequency Ω, acceleration due to vibration is $a\Omega^2\cos(\Omega t)$. Bubble departure occurs at maximum value of buoyancy force, i.e. when acceleration is $(g + a\Omega^2)$. With

$$R_B = \frac{\text{volume of bubble with vibration}}{\text{volume of bubble without vibration}} \qquad (2)$$

$$R_{B,regime\ (a)} = \frac{1}{\left(1 + \dfrac{a\Omega^2}{g}\right)} \qquad (3.a)$$

$$R_{B,regime\ (b)} = \frac{1}{\left(1 + \dfrac{a\Omega^2}{g}\right)^{3/4}} \qquad (3.b)$$

$$R_{B,regime\ (c)} = \frac{1}{\left(1 + \dfrac{a\Omega^2}{g}\right)^{3/5}} \qquad (3.c)$$

When bubble formation frequency ϕ equals vibration frequency $f = \Omega/2\pi$,

$$\dot{V} = fV_B = \frac{\Omega V_{B,vib}}{2\pi} \qquad (4)$$

$$V_{B,vibr.} = \frac{2\pi\dot{V}}{\Omega} \qquad (5)$$

Using eqs (1.a, b, c) and (2),

$$R_{B,regime\ (a),f=\phi} = \frac{2\pi\dot{V}}{K_a}\left(\frac{g}{\Omega}\right) \qquad (6a)$$

$$R_{B,regime\ (b),f=\phi} = \frac{2\pi\dot{V}^{1/4}}{K_b}\left(\frac{g^{3/4}}{\Omega}\right) \qquad (6b)$$

$$R_{B,regime\ (c),f=\phi} = \frac{2\pi}{K_c\dot{V}^{1/5}}\left(\frac{g^{3/5}}{\Omega}\right) \qquad (6c)$$

For the experimental conditions given in Barker and de Nevers' TABLE 1 (1984), regime (a) probably applies. The R_B values shown in Figure 4, of that work follow the trend of eq's (3) when $f \gg \phi$, and are covered by the range of eq's (6) when $f = \phi$ considering the data shown in TABLE 1.

A more detailed list of data than those shown by Baker and de Nevers (1984) is desirable, i.e. the values of amplitude, frequencies, flowrate, orifice diameter and fluid properties for each of the experimental points should be shown; also what happened at $(a\Omega^2/g)$ values greater than about 1.5?

With regard to consideration of industrial apparatus, the orifice velocities used, i.e. up to 0.06 ms^{-1} on the basis of given flowrates (TABLE 1) are two orders of magnitude smaller than those used in industrial sieve trays. Are there any data regarding the effect of vibration on fluid behavior on such trays?

NOTATION:

a = amplitude of vibration L
f = vibration frequency t^{-1}
g = gravity acceleration, Lt^{-2}
K_a = constant for system L^4t^{-2}
K_b = constant for system $L^{1.5}t^{-0.75}$
K_c = constant for system

$$R_B = \left(\frac{\text{volume of bubble with vibration}}{\text{volume of bubble without vibration}}\right)$$

V_B = volume of bubble, L^3
$\dot{V}$ = gas flowrate, L^3t^{-1}

Greek Letters

ϕ = bubble formation frequency, t^{-1}
Ω = angular velocity, rad. t^{-1}

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Barker, C. T. and N. de Nevers, "Bubble Formation at Vibrated Orifices: Medium Chamber Volume," AIChE J., **30**, 37 (1984).
Clift, R., Grace, J. R. and Weber, M. E., "Bubbles, Drops, and Particles," Academic Press, New York, San Francisco, London, 1978.

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Reply:

We thank Professor Lehrer for his comments.

Unfortunately, his letter is entirely based on a miscommunication between our paper and him about the region of bubble behavior treated in the paper. The title of the paper makes clear that the paper is about the "...Medium-Chamber-Volume Region". The first reference in the paper is to Park et al. (1977), and says "This entire paper concerns the 'medium chamber region' as they defined this term." Professor Lehrer's entire comment is directed toward expected results in the "small-chamber-region" as defined by Park et al.

Perhaps other readers may also not have noticed the words "medium chamber" in the title of our paper nor the above sentence showing what those words meant. So it may be appropriate to illustrate some of the differences between these two regions. As Professor Lehrer points out, in the small chamber region (for the non-vibrated case), the bubble volume is independent of chamber volume and depends on the acceleration of gravity to the minus 1, minus ¾, or minus ⅗ power, depending on gas flow rates and liquid viscosity. In contrast, as shown both theoretically and experimentally by Park et al. for the non-vibrated case, in the medium chamber region, the bubble volume is independent of gravity and is linearly proportional to the chamber volume.

To illustrate how this difference carries over when we add vibrations to the list of independent variables, Professor Lehrer correctly shows that in the small chamber region, we would expect the bubble to release when the buoyant force is largest—at the bottom of the vibratory cycle. In contrast, as our paper shows theoretically and experimentally (see figure 7), in the medium chamber region with sinusoidal vibrations bubble release occurs when the liquid pressure is lowest, which happens to be the time when the buoyant force is smallest—at the top of the vibratory cycle.

In a previous paper (Barker and de Nevers 1982), we show the theoretical and experimental results for a vibrated system in the region corresponding to Professor Lehrer's "region a" and his equations 1.a and 3.a. In order to observe this region experimentally in a vibrated situation, it is necessary to take strenuous measures to guarantee that there is a constant gas flow rate to the bubble-forming orifice, with negligible chamber volume between the device which produces that constant gas flow and the bubble-forming orifice. In effect, the small-chamber-volume region must become a zero-chamber-volume region. The experimental results do indeed agree with his equation 3.a (which is the ratio of equations 1 and 2 of our previous paper) for high Reynolds numbers. For low Reynolds numbers, the experimental results are quite different. In the work reported in our previous paper, we did not explore the regions corresponding to his regions b and c.

We agree with Professor Lehrer that a more detailed listing of the experimental conditions is desirable. That listing is given in Craig Barker's Ph.D. Thesis from the Department of Chemical Engineering at the University of Utah, which is available through University Microfilms, Ann Arbor, Michigan and is listed as a reference in our paper.

We agree with Professor Lehrer that the velocities are smaller than those normally encountered in sieve trays. We do not have nor do we present data at velocities higher than are shown in the paper and in Barker's thesis.

We would like to use this opportunity to again notify readers that the senior author of the paper is Craig T. Barker. His name was misprinted as Baker in the article.*

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* See Erratum *AIChE J.* **30**, 528 (1984).

LITERATURE CITED:

Park Yongjue, A. L. Tyler, and N. de Nevers, "The Chamber Orifice Interaction in the Formation of Bubbles," *Chem. Eng. Sci.*, **32**, 907 (1977).

Barker, C. T., and N. de Nevers, "Bubble Formation in a Vertically Vibrated System—Tate's Law Region," *AIChE J.* **28**, 851 (1982).

TABLE 1

β	DVCPR	COLSYS	Chang	Lakshmanan
1.0	0.275852(−1)	0.275852(−1)	0.275852(−1)	0.275801(−1)
0.1	0.211184(−1)	0.211184(−1)	0.211184(−1)	0.211169(−1)
0.01	0.215370(−1)	0.215364(−1)	0.215369(−1)	0.215358(−1)

To the Editor:

A problem of increasing importance is that of determining the effectiveness factor for the Michaelis-Menten-type kinetics with high Thiele moduli, the governing equation for which is:

$$x^{1-s}(x^{s-1}y')' = \phi^2 y/(\beta + y)$$

subject to the boundary conditions

$$y'(0) = 0, \quad y'(1) = Sh[1 - y(1)],$$

where ϕ is the Thiele modulus; Sh is the Sherwood number; β is a reaction rate parameter; and s is a geometric factor, with s = 1,2,3, for rectangular, cylindrical, spherical geometries, respectively, and the prime denotes differentiation with respect to x. For the solution of this problem, Chang (1982) and Lakshmanan (1983) recently discussed in this journal two methods, neither of which is widely applicable nor implemented in a user-oriented, general—or even special—purpose software package. (See also Ghim and Chang, 1983.)

The purpose of this letter is to draw attention to two widely available, robust, general purpose software packages for the solution of boundary value problems for ordinary differential equations (BVODE's). The use of these in various areas of engineering has resulted in the solution of problems that were difficult and/or costly to solve, if not intractable, using old methods, and has in general removed a great deal of the guesswork from the problem of solving BVODE's.

The first of these packages, COLSYS (Ascher et al., 1981a,b), implements the method of spline collocation at Gaussian points. In this code, the desired solution is approximated by piecewise polynomials represented in terms of a B-spline basis and the problem is solved on a sequence of meshes until a user-specified set of tolerances is satisfied. The second package, DVCPR (1982), a modified version of DD04AD (1978) and the latest version of the code PASVA3 (Pereyra, 1979), implements a variable-order finite difference method based on the trapezoidal rule with deferred corrections.

In contrast to COLSYS, DVCPR requires that the problem be formulated as a first-order system. Also the same tolerance is imposed on all components of the solution, whereas COLSYS allows the user to specify a different tolerance for each component of the solution and, in fact, allows the user to impose no tolerance at all on some or all of the components. On successful termination, each code returns estimates of the errors in the components of the approximate solution.

Each of these codes was used to approximate the desired effectiveness factor from the formula

$$\eta = s(\beta + 1)y'(1)/\phi^2,$$

where the flux $y'(1)$ is determined by the codes. The results for the case $s = 3$, Sh = 100, $\phi = 100$, and different values of β are presented in Table 1, where they are compared with those obtained by Chang (1982) and Lakshmanan (1983). Chang solved the problem numerically on the interval $[\zeta,1]$ for various values of ζ, and his results given in Table 1 are those computed with $\zeta = 0.9$ or $\zeta = 0.95$. Note that these are not the values used in the comparisons given by Lakshmanan, who quoted the values computed with $\zeta = 0.99$, which are less accurate than those given in Table 1. In all of the numerical experiments reported here, a tolerance of 10^{-3} was used in DVCPR, and, in COLSYS, imposed on the flux only.

Each code offers the possibility of providing an initial approximation to the solution and an initial subdivision of the interval of definition. The given problem is easier for higher values of β; as a consequence, the results were obtained for a decreasing sequence of values of β. An initial guess of zero for both the concentration and flux (the default in each code) was used for the value $\beta = 1$, and the initial mesh supplied to the codes had points concentrated near the righthand endpoint, since steep gradients were expected near the surface of the pellet. Parameter continuation was then used to generate solutions for the other values of β; that is, for a given value of β, the initial mesh and approximation were derived from the approximate solution and final mesh determined from the previous problem.

Chang (1982) has reported that, for high Thiele moduli and small values of β, "the numerical solution becomes very difficult or fails." To demonstrate their effectiveness in such circumstances, both codes were used to solve the boundary value problem with ϕ = 1,000. No difficulties were encountered, and both codes produced identical values for the effectiveness factor as:

β	1.0	0.1	0.01
η	0.543895(−3)	0.318860(−3)	0.298956(−3)

The existence of multiple solutions of BVODE's of this type for certain values of ϕ is well known. For example, Keller (1968) showed that two solutions exist when the boundary condition at $x = 1$ is replaced by the Dirichlet condition:

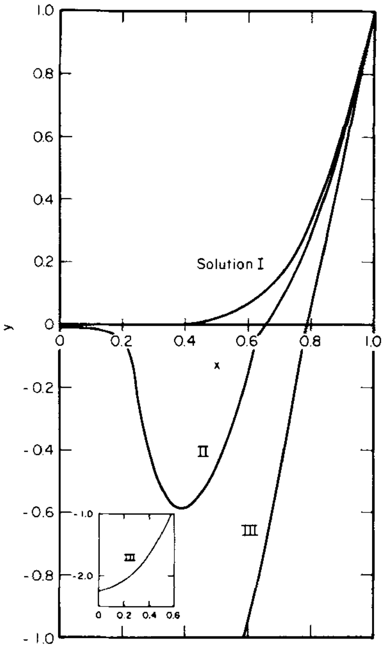

Figure 1. Three solutions to reaction/diffusion problem for Michaelis-Menten kinetics (Solutions II and III are physically impossible).

$$y(1) = 1,$$

and $\beta = 0.1$ and $\phi = \sqrt{20}$. The physical solution and the spurious, that is, not physically meaningful, solution are shown in Figure 1, labelled I and II, respectively. This problem also has a third solution, labelled III in Figure 1, which was determined by both COLSYS and DVCPR, which also reproduced the solutions determined by Keller (1968). To obtain the three solutions, different initial guesses were used, solutions I and III being computed from the initial approximations $y = y' = 0$, and $y = y' = 1$, respectively, while for solution II the approximations were taken to be

$$y = (1 - \beta/\phi^2)x^2 + \beta/\phi^2, \; y' = -2\beta x/\phi^2,$$

cf. Keller (1968). For this choice of β and ϕ, the Robin problem has also three solutions that are similar in character to those shown in Figure 1. In our numerical experiments, the spurious solution III appeared for values of ϕ as large as 1,000 in both the Dirichlet problem and the Robin problem, whereas solution II could only be determined for $\phi < 15$ in the Dirichlet case and $\phi < 10$ in the Robin problem.

The codes COLSYS and DVCPR are state-of-the-art software packages capable of handling mixed-order systems of nonlinear BVODE's, and as such are valuable tools for the numerical solution of a wide variety of problems arising in engineering.

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